

# High-density QCD and saturation of nuclear partons

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Received: 30 September 2002 /

Published online: 22 October 2003 – © Società Italiana di Fisica / Springer-Verlag 2003

**Abstract.** We review the recent finding of the two-plateau momentum distribution of sea quarks in deep inelastic scattering off nuclei in the saturation regime. The diffractive plateau which dominates for small  $\mathbf{p}$  measures precisely the momentum distribution of quarks in the beam photon, the rôle of the nucleus is simply to provide an opacity. The plateau for truly inelastic DIS exhibits a substantial nuclear broadening of the momentum distribution. Despite this nuclear broadening, the observed final-state and initial-state sea quark densities do coincide exactly. We emphasize how the saturated sea is generated from the nuclear-diluted Weizsäcker-Williams because of the anti-collinear splitting of gluons into sea quarks.

**PACS.** 11.80.La Multiple scattering – 13.87.-a Jets in large- $Q^2$  scattering – 24.85.+p Quarks, gluons, and QCD in nuclei and nuclear processes

## 1 Introduction

The saturation of nuclear partons is one of the hot issues in high-energy physics. The interpretation of nuclear opacity in terms of a fusion and saturation of nuclear partons has been introduced in 1975 [1] way before the QCD parton model: the Lorentz contraction of relativistic nuclei entails a spatial overlap of partons with  $x \lesssim x_A \approx 1/R_A m_N$  from different nucleons and the fusion of overlapping partons results in the saturation of parton densities per unit area in the impact parameter space. The pQCD link between nuclear opacity and saturation has been considered by Mueller [2] and the pQCD discussion of fusion of nuclear gluons has been revived by McLerran *et al.* [3].

In this paper we review the recent consistent derivation of nuclear parton densities in the opacity/saturation regime [4]. We demonstrate that despite the strong nuclear distortions the common wisdom that in deep inelastic scattering (DIS) the observed final-state (FS) momentum distribution of struck partons coincides with the initial-state (IS) density of partons in the probed target holds for nuclei too. The key to this derivation is the Weizsäcker-Williams (WW) glue of the relativistic nucleus as defined according to [5]. We pay a special attention to an important point that diffractive DIS in which the target nucleus does not break and is retained in the ground state, makes precisely 50 per cent of the total DIS events [6]. We point out that

the saturated diffractive plateau measures precisely the momentum distribution of (anti-)quarks in the  $q\bar{q}$  Fock state of the photon. We show how the anti-collinear splitting of WW gluons into sea quarks gives rise to nuclear saturation of the sea, despite the substantial nuclear dilution of the WW glue.

## 2 Nuclear distortions of the parton spectra

We illustrate our ideas on an example of DIS at  $x \sim x_A \ll 1$  which is dominated by interactions of  $q\bar{q}$  Fock states of the photon [6–10]. The total cross-section for interaction of the color dipole  $\mathbf{r}$  with the target nucleon equals

$$\sigma(r) = \alpha_S(r)\sigma_0 \int d^2\boldsymbol{\kappa} f(\boldsymbol{\kappa}) [1 - \exp(i\boldsymbol{\kappa}\mathbf{r})], \quad (1)$$

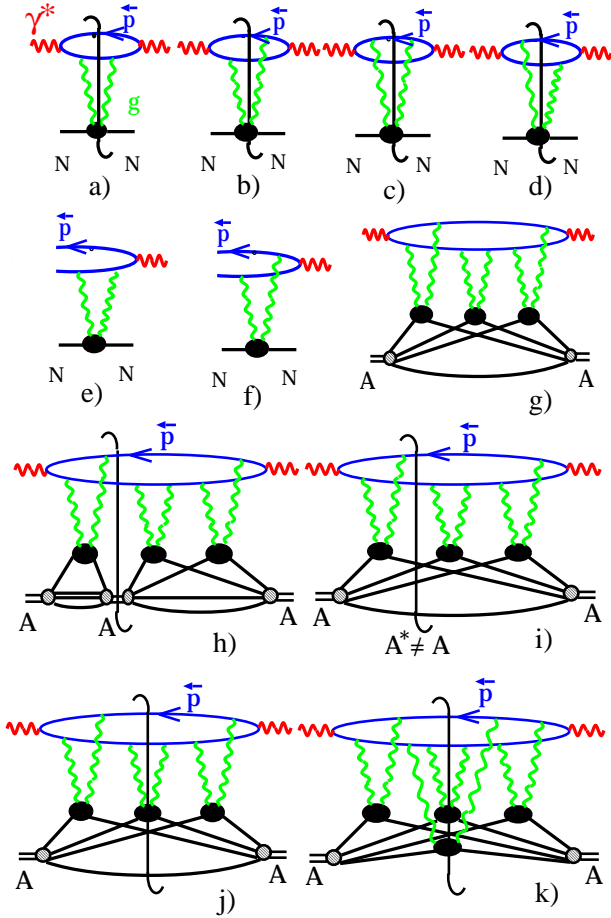
where the unintegrated glue of the target nucleon is

$$f(\boldsymbol{\kappa}) = \frac{4\pi}{N_c\sigma_0} \cdot \frac{1}{\kappa^4} \cdot \frac{\partial G}{\partial \log \kappa^2}, \quad \int d^2\boldsymbol{\kappa} f(\boldsymbol{\kappa}) = 1. \quad (2)$$

For DIS off a free nucleon target, see figs. 1a-d, the spectrum of the FS quark prior to the hadronization,

$$\frac{d\sigma_N}{d^2\mathbf{p} dz} = \frac{\sigma_0\alpha_S(\mathbf{p}^2)}{2(2\pi)^2} \int d^2\boldsymbol{\kappa} f(\boldsymbol{\kappa}) \times |\langle \gamma^* | \mathbf{p} \rangle - \langle \gamma^* | \mathbf{p} - \boldsymbol{\kappa} \rangle|^2, \quad (3)$$

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**Fig. 1.** The pQCD diagrams for inclusive (a-d) and diffractive (e, f) DIS off protons and nuclei (g-k). Diagrams (a-d) show the unitarity cuts with color excitation of the target nucleon, (g) a generic multiple-scattering diagram for Compton scattering off nucleus, (h) the unitarity cut for a coherent diffractive DIS, (i) the unitarity cut for quasielastic diffractive DIS with excitation of the nucleus  $A^*$ , (j, k) the unitarity cuts for truly inelastic DIS with single and multiple color excitation of nucleons of the nucleus.

where  $\mathbf{p}$  is the transverse momentum, and  $z$  the Feynman variable, coincides, upon the  $z$  integration, with the conventional IS unintegrated  $\mathbf{p}$  distribution of partons in the target. Notice that the target nucleon is color-excited and there is no rapidity gap in the FS.

In DIS off nuclei one must distinguish the three principal processes with distinct unitarity cuts of the forward Compton amplitude (fig. 1g): the coherent diffraction dissociation (D) of the photon (fig. 1h), quasielastic diffraction dissociation (qel) followed by excitation and breakup of the target nucleus (fig. 1i) —in both of them there is no color flow between the photon debris and the nucleus—, and the truly inelastic (in) DIS with color excitation of nucleons of the target nucleus (fig. 1j, k).

The technical trick with separation of the color dipole-nucleon  $S$ -matrix into the color-octet and color-singlet components, and the derivation of the full two-parton momentum spectrum are found in [4], here we only cite the

single-parton spectrum

$$\frac{d\sigma_{\text{in}}}{d^2\mathbf{p} dz} = \frac{1}{(2\pi)^2} \int d^2\mathbf{b} \int d^2\mathbf{r}' d^2\mathbf{r} \exp [i\mathbf{p}(\mathbf{r}' - \mathbf{r})] \times \Psi^*(\mathbf{r}') \Psi(\mathbf{r}) \times \left\{ \exp \left[ -\frac{1}{2}\sigma(\mathbf{r} - \mathbf{r}')T(\mathbf{b}) \right] - \exp \left[ -\frac{1}{2}[\sigma(\mathbf{r}) + \sigma(\mathbf{r}')]T(\mathbf{b}) \right] \right\}. \quad (4)$$

Evidently, the dependence of nuclear attenuation factors on  $\mathbf{r}, \mathbf{r}'$  shall distort strongly the observed momentum distribution of quarks.

### 3 WW glue of nuclei

In the further parton model interpretation of this spectrum we resort to the NSS representation [5]

$$\Gamma_A(\mathbf{b}, \mathbf{r}) = 1 - \exp \left[ -\frac{1}{2}\sigma(r)T(\mathbf{b}) \right] = \int d^2\boldsymbol{\kappa} \phi_{\text{WW}}(\boldsymbol{\kappa}) [1 - \exp(i\boldsymbol{\kappa}\mathbf{r})], \quad (5)$$

where, driven by a close analogy to (1), (2) in terms of  $f(\boldsymbol{\kappa})$ , we interpret

$$\phi_{\text{WW}}(\boldsymbol{\kappa}) = \sum_{j=1}^{\infty} \nu_A^j(\mathbf{b}) \cdot \frac{1}{j!} f^{(j)}(\boldsymbol{\kappa}) \exp[-\nu_A(\mathbf{b})] \quad (6)$$

as the WW unintegrated glue of a nucleus per unit area in the impact parameter plane. Here

$$\nu_A(\mathbf{b}) = \frac{1}{2} \alpha_S(r) \sigma_0 T(\mathbf{b})$$

defines the nuclear opacity and the  $j$ -fold convolutions

$$f^{(j)}(\boldsymbol{\kappa}) = \int \prod_{i=1}^j d^2\boldsymbol{\kappa}_i f(\boldsymbol{\kappa}_i) \delta\left(\boldsymbol{\kappa} - \sum_{i=1}^j \boldsymbol{\kappa}_i\right) \quad (7)$$

describe the contribution to the diffractive amplitudes from  $j$  split pomerons [5].

A discussion of the nuclear antishadowing property of the hard WW glue is found in [5]. A somewhat involved analysis of the properties of the convolutions (7) in the soft region shows that they develop a plateau-like behaviour with the width of the plateau which expands  $\propto j$ . The gross features of the WW nuclear glue in the soft region are well reproduced by

$$\phi_{\text{WW}}(\boldsymbol{\kappa}) \approx \frac{1}{\pi} \frac{Q_A^2}{(\boldsymbol{\kappa}^2 + Q_A^2)^2}, \quad (8)$$

where the saturation scale  $Q_A^2 = \nu_A(\mathbf{b}) Q_0^2 \propto A^{1/3}$ . Notice the nuclear dilution of soft WW glue,  $\phi_{\text{WW}}(\boldsymbol{\kappa}) \propto 1/Q_A^2 \propto A^{-1/3}$ . The soft parameters  $Q_0^2$  and  $\sigma_0$  are related to the integrated glue of the proton in the soft region:

$$Q_0^2 \sigma_0 \sim \frac{2\pi^2}{N_c} G_{\text{soft}}, \quad G_{\text{soft}} \sim 1.$$

## 4 An exact equality of the IS and FS parton densities

Making use of the NSS representation, the total nuclear photoabsorption cross-section can be cast in the form

$$\sigma_A = \int d^2\mathbf{b} \int dz \int \frac{d^2\mathbf{p}}{(2\pi)^2} \times \int d^2\boldsymbol{\kappa} \phi_{\text{WW}}(\boldsymbol{\kappa}) |\langle \gamma^* | \mathbf{p} \rangle - \langle \gamma^* | \mathbf{p} - \boldsymbol{\kappa} \rangle|^2, \quad (9)$$

which has a profound semblance to (3) and one is tempted to take the differential form of (9) as a definition of the IS sea quark density in a nucleus:

$$\frac{d\bar{q}_{\text{IS}}}{d^2\mathbf{b} d^2\mathbf{p}} = \frac{1}{2} \frac{Q^2}{4\pi^2 \alpha_{\text{em}}} \frac{d\sigma_A}{d^2\mathbf{b} d^2\mathbf{p}}. \quad (10)$$

Remarkably, in terms of the NSS-defined WW nuclear glue, all intranuclear multiple-scattering diagrams of fig. 1g sum up to precisely the same four diagrams fig. 1a-d as in DIS off free nucleons. Although  $\mathbf{p}$  emerges here just as a formal Fourier parameter, we shall demonstrate that it can be identified with the momentum of the observed final state antiquark.

Indeed, making use of the NSS representation, after some algebra one finds

$$\frac{d\sigma_{\text{in}}}{d^2\mathbf{b} d^2\mathbf{p} dz} = \frac{1}{(2\pi)^2} \times \left\{ \int d^2\boldsymbol{\kappa} \phi_{\text{WW}}(\boldsymbol{\kappa}) |\langle \gamma^* | \mathbf{p} \rangle - \langle \gamma^* | \mathbf{p} - \boldsymbol{\kappa} \rangle|^2 - \left| \int d^2\boldsymbol{\kappa} \phi_{\text{WW}}(\boldsymbol{\kappa}) (\langle \gamma^* | \mathbf{p} \rangle - \langle \gamma^* | \mathbf{p} - \boldsymbol{\kappa} \rangle) \right|^2 \right\}, \quad (11)$$

$$\frac{d\sigma_{\text{D}}}{d^2\mathbf{b} d^2\mathbf{p} dz} = \frac{1}{(2\pi)^2} \times \left| \int d^2\boldsymbol{\kappa} \phi_{\text{WW}}(\boldsymbol{\kappa}) (\langle \gamma^* | \mathbf{p} \rangle - \langle \gamma^* | \mathbf{p} - \boldsymbol{\kappa} \rangle) \right|^2. \quad (12)$$

As far as diffraction is concerned, the analogy between (12) and its counterpart for free nucleons [5,8,11], and nuclear WW glue  $\phi_{\text{WW}}(\boldsymbol{\kappa})$  and  $f(\boldsymbol{\kappa})$  thereof, is complete. Putting the inelastic and diffractive components of the FS quark spectrum together, we evidently find the FS parton density which exactly coincides with the IS parton density (10) such that  $\mathbf{p}$  is indeed the transverse momentum of the FS sea quark. The interpretation of this finding is not trivial, though.

## 5 The two-plateau spectrum of sea quarks

Consider first the domain of  $\mathbf{p}^2 \lesssim Q^2 \lesssim Q_A^2$  such that the nucleus is opaque for all color dipoles in the photon. Hereafter we assume that the saturation scale  $Q_A^2$  is so large that  $\mathbf{p}^2, Q^2$  are in the pQCD domain and neglect

the quark masses. In this regime the nuclear counterparts of the crossing diagrams of figs. 1b, d, f can be neglected. Then, in the classification of [5], diffraction will be dominated by the contribution from the Landau-Pomeranchuk diagram of fig. 1e with the result

$$\frac{d\bar{q}_{\text{FS}}}{d^2\mathbf{b} d^2\mathbf{p}} \Big|_{\text{D}} \approx \frac{1}{2} \frac{Q^2}{4\pi^2 \alpha_{\text{em}}} \int dz \times \left| \int d^2\boldsymbol{\kappa} \phi_{\text{WW}}(\boldsymbol{\kappa}) |\langle \gamma^* | \mathbf{p} \rangle|^2 \right. \approx \frac{N_c}{4\pi^4}. \quad (13)$$

Remarkably, diffractive DIS measures the momentum distribution of quarks and antiquarks in the  $q\bar{q}$  Fock state of the photon. This result, typical of the Landau-Pomeranchuk mechanism, is a completely generic one and would hold for any beam particle such that its coupling to colored partons is weak. In contrast to diffraction off free nucleons [8,11,12], diffraction off opaque nuclei is dominated by the anti-collinear splitting of hard gluons into soft sea quarks,  $\boldsymbol{\kappa}^2 \gg \mathbf{p}^2$ . Precisely for this reason one finds the saturated FS quark density, because the nuclear dilution of the WW glue is compensated for by the expanding plateau. The result (13) has no counterpart in DIS off free nucleons because diffractive DIS off free nucleons is negligibly small even at HERA,  $\eta_{\text{D}} \lesssim 6\text{--}10\%$ .

The related analysis of the FS quark density for truly inelastic DIS in the same domain of  $\mathbf{p}^2 \lesssim Q^2 \lesssim Q_A^2$  gives

$$\frac{d\bar{q}_{\text{FS}}}{d^2\mathbf{b} d^2\mathbf{p}} \Big|_{\text{in}} = \frac{1}{2} \frac{Q^2}{4\pi^2 \alpha_{\text{em}}} \int dz \times \int d^2\boldsymbol{\kappa} \phi_{\text{WW}}(\boldsymbol{\kappa}) |\langle \gamma^* | \mathbf{p} - \boldsymbol{\kappa} \rangle|^2 = \frac{Q^2}{8\pi^2 \alpha_{\text{em}}} \phi_{\text{WW}}(0) \int^{Q^2} d^2\boldsymbol{\kappa} \int dz |\langle \gamma^* | \boldsymbol{\kappa} \rangle|^2 \approx \frac{N_c}{4\pi^4} \frac{Q^2}{Q_A^2} \theta(Q_A^2 - \mathbf{p}^2). \quad (14)$$

It describes final states with color excitation of a nucleus, but as a function of the photon wave function and nuclear WW gluon distribution it is completely different from (3) for free nucleons. The  $\theta$ -function simply indicates that the plateau for inelastic DIS extends up to  $\mathbf{p}^2 \lesssim Q_A^2$ . For  $Q^2 \ll Q_A^2$  the inelastic plateau contributes little to the transverse-momentum distribution of soft quarks,  $\mathbf{p}^2 \lesssim Q^2$ , but the inelastic plateau extends way beyond  $Q^2$  and its integral contribution to the spectrum of FS quarks is exactly equal to that from diffractive DIS. Such a two-plateau structure of the FS quark spectrum is a new finding and has not been considered before.

Now notice, that in the opacity regime the diffractive FS parton density coincides with the contribution  $\propto |\langle \gamma^* | \mathbf{p} \rangle|^2$  to the IS sea parton density from the spectator diagram 1a, whereas the FS parton density for truly inelastic DIS coincides with the contribution to IS sea partons from the diagram of fig. 1c. The contribution from the crossing diagrams 1b, d is negligibly small.

Our results (13) and (14), especially nuclear broadening and unusually strong  $Q^2$ -dependence of the FS/IS parton density from truly inelastic DIS, demonstrate clearly

a distinction between diffractive and inelastic DIS. Our considerations can readily be extended to the spectrum of soft quarks,  $\mathbf{p}^2 \lesssim Q_A^2$ , in hard photons,  $Q^2 \gtrsim Q_A^2$ . In this case the result (13) for diffractive DIS is retained, whereas in the numerator of the result (14) for truly inelastic DIS one must substitute  $Q^2 \rightarrow Q_A^2$ , so that in this case  $dq_{\text{FS|D}} \approx dq_{\text{FS|in}}$  and  $dq_{\text{IS}} \approx 2dq_{\text{FS|D}}$ . The evolution of soft nuclear sea,  $\mathbf{p}^2 \lesssim Q_A^2$ , is entirely driven by an anti-collinear splitting of the NSS-defined WW nuclear glue into the sea partons.

The early discussion of the FS quark density in the saturation regime is due to Mueller [13]. Mueller focused on  $Q^2 \gg Q_A^2$  and discussed neither a distinction between diffractive and truly inelastic DIS nor a  $Q^2$ -dependence and broadening (8) for truly inelastic DIS at  $Q^2 \lesssim Q_A^2$ .

## 6 More signals of saturation in diffractive DIS

The flat  $\mathbf{p}^2$  distribution of forward  $q, \bar{q}$  jets in truly inelastic DIS in the saturation regime must be contrasted to the  $\propto G(\mathbf{p}^2)/\mathbf{p}^2$  spectrum for the free nucleon target. In the diffractive DIS the saturation gain is much more dramatic: flat  $\mathbf{p}^2$  distribution of forward  $q, \bar{q}$  jets in diffractive DIS in the saturation regime must be contrasted to the  $\propto 1/(\mathbf{p}^2)^2$  spectrum for the free nucleon target [8, 11]. In the exclusive diffractive DIS, *i.e.*, the vector meson production, the relevant hard scale for the proton target equals  $\bar{Q}^2 \approx \frac{1}{4}(Q^2 + m_V^2)$  and the transverse cross-section has been predicted to behave as [14]

$$\sigma_T \propto G^2(x, \bar{Q}^2)(\bar{Q}^2)^{-4}. \quad (15)$$

At  $\bar{Q}^2 > Q_A^2$  the same would hold for nuclei too, but in the opposite case of  $\bar{Q}^2 < Q_A^2$  the  $\bar{Q}^2$ -dependence is predicted to change to

$$\sigma_T \propto G^2(x, \bar{Q}^2)(Q_A^2)^{-2}(\bar{Q}^2)^{-2}. \quad (16)$$

## 7 Conclusions

We reported a derivation of the FS parton spectrum. Our result (4) summarizes in an elegant way intranuclear distortions due to multiple diffractive rescatterings and color excitations of the target nucleus. In conjunction with the NSS definition of the WW glue of the nucleus, eqs. (5), (6), it gives an explicit form of the FS parton densities. The two-plateau FS quark density with the strong  $Q^2$ -dependence of the plateau for truly inelastic DIS has not been discussed before. A comparison with the IS nuclear parton densities which evolve from the NSS-defined WW nuclear glue shows an exact equality of the FS and IS parton densities. The plateau-like saturated nuclear quark density is suggestive of the Fermi statistics, but our principal point that for any projectile which interacts weakly with colored partons the saturated density measures the momentum distribution in the  $q\bar{q}, gg, \dots$  Fock state of the projectile disproves the Fermi-statistics interpretation. The spin and color multiplet of colored partons

the photon couples to is completely irrelevant, what only counts is an opacity of heavy nuclei. The anti-collinear splitting of WW nuclear glue into soft sea partons is a noteworthy feature of the both diffractive DIS and IS sea parton distributions. The emergence of a saturated density of IS sea partons from the nuclear-diluted WW glue is due to the nuclear broadening of the plateau (8). Because the predominance of diffraction is a very special feature of DIS [6], one must be careful with applying the IS parton densities to, for instance, nuclear collisions, in which diffraction would not be of any significance.

One can go one step further and consider interactions with the opaque nucleus of the  $q\bar{q}g$  Fock states of the photon. Then the above analysis can be extended to  $x \ll x_A$  and the issue of the  $x$ -dependence of the saturation scale  $Q_A^2$  can be addressed following the discussion in [9]. We only mention here that as far as diffraction and IS parton densities are concerned, the NSS-defined WW glue remains a useful concept and the close correspondence between  $\phi_{\text{WW}}(\boldsymbol{\kappa})$  for the nucleus and  $f(\boldsymbol{\kappa})$  for the nucleon is retained.

This work has been partly supported by DAAD and Nordita, the INTAS grants 97-30494 & 00-00366 and DFG grant 436RUS17/119/01.

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